TOPOLOGY 2
BACKPAPER EXAMINATION

This exam is of **50 marks**. Please **read all the questions carefully** and **do not cheat**. Please feel free to use whatever theorems you have learned in class after stating them clearly. Please **hand in your phones** at the beginning of the class. Please sign the following statement:

I have not used any unfair or illegal means to answer any of the questions in this exam. I have answered this paper honestly and truthfully.

Name: **Signature:**

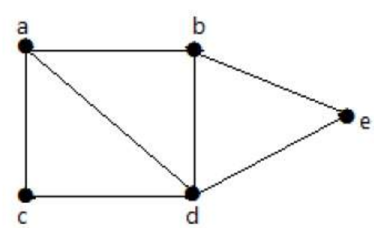
1a. Give an example of a pointed space (X, P) such that the natural map

$$\pi_1(X; P) \longrightarrow H_1(X; \mathbb{Z})$$

is **not** surjective. (4)

1b. When is it surjective and what is the relationship between the two in that case? (4)

2. Consider the following graph X –



2a. Compute the fundamental group based at the point b, $\pi_1(X; b)$. (4)

2b. Compute the homology groups $H_i(X; \mathbb{Z})$ for all i (8)

3a. State and prove the **Long Exact Sequence for Relative Homology groups**. (8)

3b. Let E^2 denote the disc in $\mathbb{R}^2$ and S^1 its boundary circle. Compute (7)

$$H_2(E^2, S^1; \mathbb{Z})$$

3c. Let S^3 denote the 3-sphere and D^2 a 2-disc on the sphere. What is $H_1(S^3 - D^2; \mathbb{Z})$? You may use any theorem done in class - just state the theorem precisely. (5)

4. Prove the **Barratt-Whitehead Lemma** – Given a diagram of R-modules and homomorphisms such that all the rectangles commute and the rows are exact (10)

$$\begin{array}{ccccccccccccc}
\cdots & \longrightarrow & C_{i+1} & \xrightarrow{h_{i+1}} & A_i & \xrightarrow{f_i} & B_i & \xrightarrow{g_i} & C_i & \xrightarrow{h_i} & A_{i-1} & \longrightarrow & \cdots \\
 & & \downarrow{\scriptstyle \gamma_{i+1}} & & \downarrow{\scriptstyle \alpha_i} & & \downarrow{\scriptstyle \beta_i} & & \downarrow{\scriptstyle \gamma_i} & & \downarrow{\scriptstyle \alpha_{i-1}} & & \\
\cdots & \longrightarrow & C'_{i+1} & \xrightarrow{h'_{i+1}} & A'_i & \xrightarrow{f'_i} & B'_i & \xrightarrow{g'_i} & C'_i & \xrightarrow{h'_i} & A'_{i-1} & \longrightarrow & \cdots
\end{array}$$

If the γ_i are isomorphisms then show that there is a **long exact sequence**

$$\cdots \longrightarrow A_i \xrightarrow{\Phi_i} A'_i \oplus B_i \xrightarrow{\Psi_i} B'_i \xrightarrow{\Gamma_i} A_{i-1} \longrightarrow \cdots$$

where $\Phi_i = (\alpha_i \oplus f_i) \circ \Delta$, $\Psi_i = \nabla \circ (-f'_i \oplus \beta_i)$ and $\Gamma_i = h_i \circ \gamma_i^{-1} \circ g'_i$ and $\Delta(a) = (a, a)$, $\nabla(x, y) = x + y$.